Outline for section 1

Quantum computers – introduction

Postulates of quantum mechanics
- Quantum state
- Evolution of quantum systems
- Quantum measurement
- Composition of quantum systems

Quantum machine learning with quantum circuits
- Machine learning and information encoding
- Simple application of gate model for quantum classification problem

Adiabatic quantum computing
- Basics
- D-Wave annealer
- Application in machine learning

Quantum APIs

Summary
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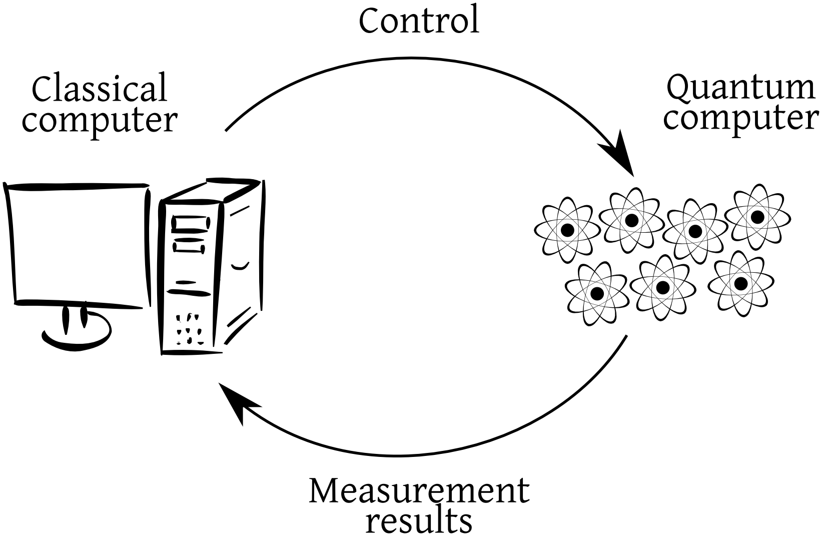
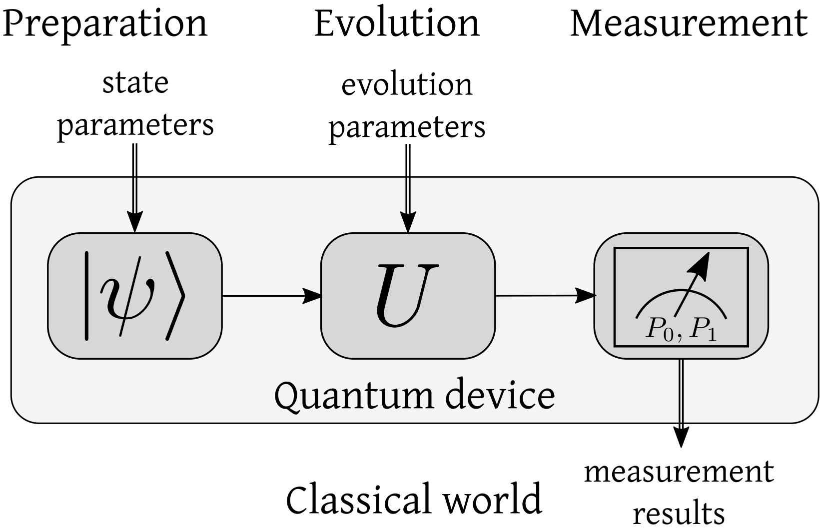
Quantum computation control loop



Computation as experiment



Outline for section 2

Quantum computers – introduction
Postulates of quantum mechanics
Quantum state
Evolution of quantum systems
Quantum measurement
Composition of quantum systems
Quantum machine learning with quantum circuits
Machine learning and information encoding
Simple application of gate model for quantum classification problem
Adiabatic quantum computing
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Quantum state I

We recall the postulates of quantum mechanics, upon which the proposed method is derived.

- The **state** of quantum system is represented by a **complex unit vector** from n-dimensional Euclidean vector space $\mathbb{C}^n$.
- Let us introduce an orthonormal complete set of vectors (computational basis)

$$|0\rangle = \begin{bmatrix} 1 \\ \vdots \\ 0 \end{bmatrix}, \ldots, \ |n-1\rangle = \begin{bmatrix} 0 \\ \vdots \\ 1 \end{bmatrix}. \tag{1}$$

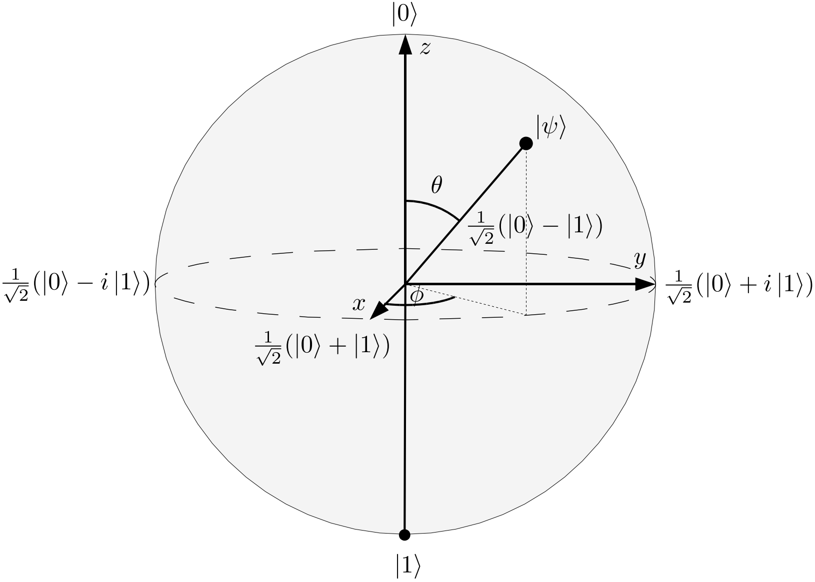
Quantum state II

- The $|x\rangle$ vector is called '**ket**' and its Hermitian conjugation $(|x\rangle)^\dagger = \langle x|$ is called '**bra**'.
- We can represent any valid state of a n-level quantum state $|\psi\rangle$ as normalized **linear combination** of the basis vectors:

$$|\psi\rangle = \alpha_1 |0\rangle + \dots + \alpha_n |n-1\rangle, \tag{2}$$

  where $\alpha_1, \dots, \alpha_n \in \mathbb{C}$ and $\sum_{i=1}^{n} |\alpha_i|^2 = 1$.
- We assume that two vectors $|\psi\rangle$ and $|\phi\rangle$ represent the same physical state if $|\psi\rangle = e^{\mathrm{i}\theta} |\phi\rangle$, for $\theta \in \mathbb{R}$.

Qubit, Bloch sphere



Time evolution of a quantum system I

- The evolution of quantum systems is governed by the **Schrödinger equation**

$$\frac{\mathrm{d}\,|\psi\rangle}{\mathrm{d}t} = -\mathrm{i}H\,|\psi\rangle\,, \tag{3}$$

  where H is a hermitian operator *i.e.* $H = H^\dagger$ called **Hamiltonian** of the system. Here we put Planck constant equal to one.

- Solutions of the (3) are given by

$$|\psi_{t_1}\rangle = U(t_0, t_1)\,|\psi_{t_0}\rangle\,, \tag{4}$$

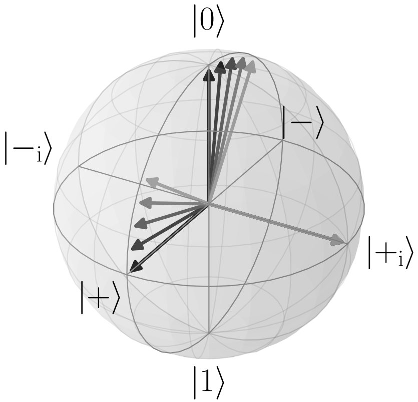
  where $|\psi_{t_0}\rangle$ is the initial state of the system, $|\psi_{t_1}\rangle$ is final state of the system, and $U(t_0, t_1)$ is **unitary operator** driving the system from time t_0 to t_1.

Time evolution of a quantum system II

- An operator is unitary iff $UU^\dagger = U^\dagger U = \mathbb{1}$
- Assuming that Hamiltonian H is time is constant between time t_0 and t_1 $U(t_0, t_1)$ can be obtained from equation

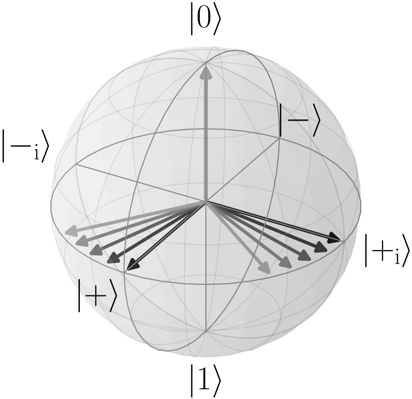
$$U(t_0, t_1) = e^{-\mathrm{i}(t_1 - t_0)H}. \tag{5}$$

Unitary gates, quantum evolution



Rotation around y-axis

Unitary gates, quantum evolution



Rotation around z-axis

Measurement

- In order to measure the state of a quantum system one has to chose a **quantum measurement** that is a function μ from finite set of measurements outcomes $A = \{a_i\}_{i=1}^{n}$ to set the of projection operators $P = \{P_i\}_{i=1}^{n}$ such that

$$\sum_{i=1}^{n} P_i = \mathbb{1} \text{ and } P_i^2 = P_i. \tag{6}$$

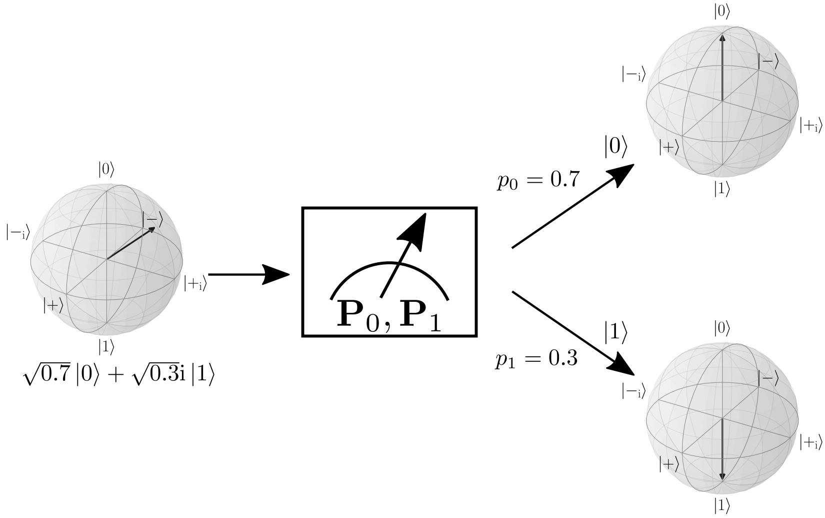
- The **probability of measuring outcome** a_i given the quantum system in in state $|\psi\rangle$ is

$$p(a_i) = \langle\psi| P_i |\psi\rangle\,. \tag{7}$$

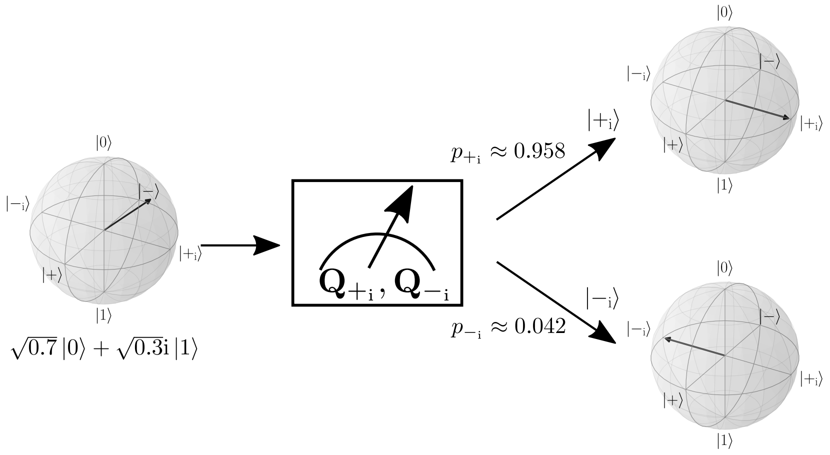
  The state of the quantum system **after the measurement** outcome a_i was obtained becomes

$$|\psi\rangle_{a_i} = \frac{P_i |\psi\rangle}{\sqrt{\langle\psi| P_i |\psi\rangle}}. \tag{8}$$

Measurement



Measurement



Composition of quantum systems

The operation which allows us to join two independent quantum systems is the **tensor product**. Lets take **two qubit** states

$$|\psi\rangle = \begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} = \psi_0 |0\rangle + \psi_1 |1\rangle ,$$

$$|\phi\rangle = \begin{bmatrix} \phi_0 \\ \phi_1 \end{bmatrix} = \phi_0 |0\rangle + \phi_1 |1\rangle , \tag{9}$$

then we can write their joint state in $\mathbb{C}^2 \otimes \mathbb{C}^2$ as

$$|\psi\rangle \otimes |\phi\rangle = \begin{bmatrix} \psi_0\phi_0 \\ \psi_0\phi_1 \\ \psi_1\phi_0 \\ \psi_1\phi_1 \end{bmatrix} . \tag{10}$$

Entanglement

Bell state

$$
\begin{aligned}
\left|\Phi^+\right\rangle =& \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle) \stackrel{?}{=} \\
\stackrel{?}{=}& \psi_0\phi_0 |0\rangle \otimes |0\rangle + \psi_0\phi_1 |0\rangle \otimes |1\rangle + \psi_1\phi_0 |1\rangle \otimes |0\rangle + \psi_1\phi_1 |1\rangle \otimes |1\rangle
\end{aligned}
$$

Contradiction

$$
\begin{aligned}
\psi_0\phi_0 = \psi_1\phi_1 = \frac{1}{\sqrt{2}}, \\
\psi_0\phi_1 = \psi_1\phi_0 = 0.
\end{aligned}
$$

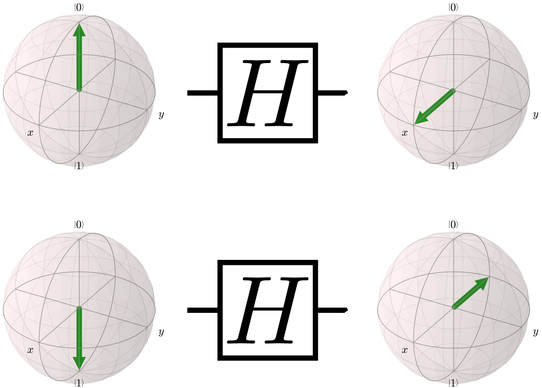
Gate model of quantum computation I

UNITARY | $-\boxed{U}-$ | $U|\psi_1\rangle = |\psi_2\rangle$, $U^\dagger|\psi_2\rangle = |\psi_1\rangle$

$$U = e^{i\varphi/2}\begin{bmatrix} e^{i\psi} & 0 \\ 0 & e^{-i\psi} \end{bmatrix}\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}\begin{bmatrix} e^{i\Delta} & 0 \\ 0 & e^{-i\Delta} \end{bmatrix}$$

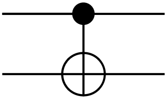
Gate model of quantum computation II

HADAMARD | $-\boxed{H}-$ | $|0\rangle \mapsto \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|1\rangle \mapsto \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$



Gate model of quantum computation III

CNOT



in_1	in_2	out_1	out_2
0	0	0	0
0	1	0	1
1	0	1	1
1	1	1	0

Gate model of quantum computation IV

$$|0\rangle \otimes |0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |0\rangle \rightarrow \frac{1}{\sqrt{2}}(|0\rangle \otimes |0\rangle + |1\rangle \otimes |1\rangle)$$

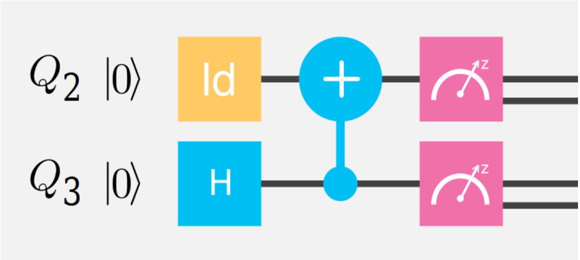


Figure: Bell state preparation circuit.

Gate model of quantum computation V

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Results date: May 9, 2016 1:53:17 PM

Number of shots: 1024

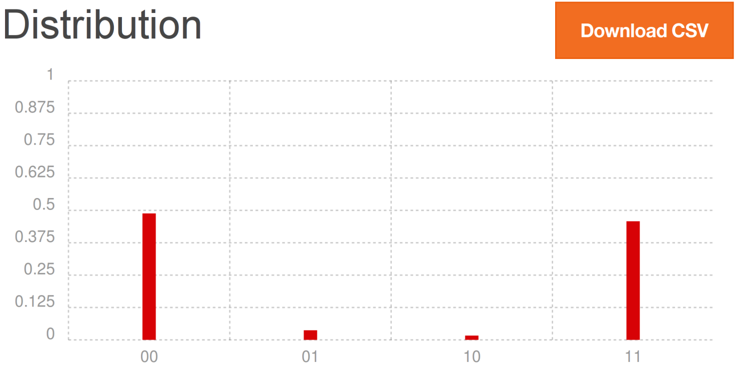

Figure: Histogram of measurements obtained from IBM 5Q machine.

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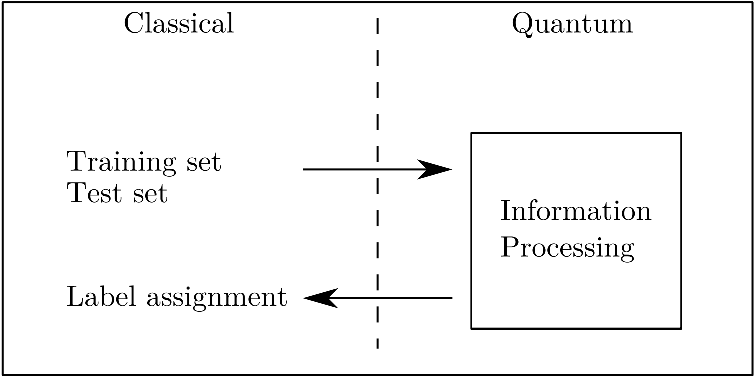
Quantum classification algorithm

- Input data is classical.
- Output data is classical.
- Information processing is quantum.



Starting point

Schuld, Maria, Mark Fingerhuth, and Francesco Petruccione. "Implementing a distance-based classifier with a quantum interference circuit." EPL (Europhysics Letters) 119.6 (2017): 60002.

- A quantum algorithm that recovers classical classification algorithm.
- A set of quantum basis states is a product of features number, classes number and training set size.